REAL ESTATE MATHEMATICS

IRR NPV Return on Equity
MIRR FMRR CAP RATE

By
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Potential Gross Income Multiplier = Market Price / Potential Gross Income

Potential Gross Income (PGI) – Page 8

PGI = Potential Gross Rental Income (PGRI) + Other Income

PGRI = Net Leasable Area or No. of Units * Market Rent (per sq. ft. or per unit)

Effective Gross Income Multiplier (EGIM) – Page 9

Effective Gross Income Multiplier = Market Price / Effective Gross Income

Effective Gross Income (EGI) – Page 9

EGI = Potential Gross Rental Income + Other Income – Vacancy & Bad Debt Allowance

Net Income Multiplier (NIM) – Page 10

NIM = Market Price / Net Operating Income (NOI)

Net Operating Income (NOI) – Page 10

NOI = Effective Gross Income – Operating Expenses + Recoveries

Overall Capitalization Rate or Income Return or All-Risks Yield – Page 11

Capitalization Rate or Income Return = Net Operating Income (NOI) / Market Price
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Terminal Capitalization Rate or Reversion Yield or Exit Cap Rate – Page 22

Terminal Capitalization Rate = \( \frac{\text{NOI}(n)}{\text{Expected Resale Price}(n)} \)

Appreciation Return or Capital Growth – Page 23

Appreciation Return or Capital Growth = \( \frac{(V_t - \text{Sales Cost} - V_{t-1})}{V_t} \)

or equivalently

Appreciation Return or Capital Growth = \( \left[ \frac{(V_t - \text{Sales Cost})}{V_{t-1}} \right] - 1 \)

Total Return – Page 24

Total Return = Income Return + Appreciation Return

or

Total Return = \( \frac{(\text{NOI} + V_t - \text{Sales Cost} - V_{t-1})}{V_{t-1}} \)

Return on Total Capital (ROR) – Page 25

ROR = \( \frac{\text{NOI}}{\text{Total Capital Invested}} \)

Return on Equity (ROE) / Cash-on-Cash Return / Cash Flow Rate/Equity Dividend Rate/ Equity Capitalization Rate – Page 25

ROE = Before Tax Equity Cash Flow (BTECF) / Equity Investment

Before Tax Equity Cash Flow (BTECF) – Page 26

BTECF = NOI – Debt Service

Equity Investment – Page 26

Equity Investment = Investment Cost – Loan Amount

Debt Service – Page 26
Debt Service = Mortgage Constant * Loan Amount

Loan Amount = Loan-to-Value-Ratio * Purchase Price \((13)\)

Mortgage Constant = \(i / [ 1- (1/(1+i)^n)]\)

Payback Period = Equity Investment Cost / Annual After Tax Cash Flow

ATCF = NOI – (Taxable Income * Tax Rate) – Debt Service

Taxable Income = NOI – Depreciation – Interest Payment

Breakeven Occupancy = (Operating Expenses + Debt Service)/ Potential Gross Income

NPV = \(CF_0 + CF_1/(1+IRR) + CF_2/(1+IRR)^2 + \ldots + CF_n/(1+IRR)^n = 0\)

IRR Formula for One Period

\(IRR = \left[ \frac{CF_1}{-CF_0} \right] - 1\)

Income Tax Payment in Association with Income Producing Property

Tax payment = (NOI – Depreciation- Interest Payment) * Tax Rate
Cash Flow for Last Period of Analysis \( n \) – Page 35

\[
CF_n = ATCF + \text{Resale Price}_n - \text{Capital Gains Tax} - \text{Outstanding Loan Balance}
\]

\[\text{Capital Gains Tax} = [\text{Resale Price}_n - \text{Selling Expenses} - \text{Cost Basis}] \times \text{Capital Gains Tax Rate}\]

Resale Price in Period \( n \) – Page 36

\[
\text{Resale Price}_n = \frac{\text{NOI}_n}{\text{Capitalization Rate}_n}
\]

Annual Rental Income of Occupied Multi-Tenant Property – Page 37

\[
\text{Rental Income}_t = A_1 \times R_{1t} \times 12 + A_2 \times R_{2t} \times 12 + \ldots + A_i \times R_{it} \times 12
\]

One-Period Lease Rate Growth Formula – Page 38

\[
R_{t+1} = R_t \times [1 + \text{CPI}_t + D]
\]

Multi-Period Lease Rate Growth Formula with Intertemporally Variable CPI Forecast – Page 39

\[
R_{t+n} = R_t \times [1 + \text{CPI}_t + D] \times [1 + \text{CPI}_{t+1} + D] \times \ldots \times [1 + \text{CPI}_{t+n-1} + D]
\]

Multi-period Lease Rate Growth Formula with Constant CPI Forecast – Page 39

\[
R_{t+n} = R_t \times [1 + \text{CPI}_t + D] \times [1 + \text{CPI}_{t+1} + D]^{n-1}
\]

Modified IRR (MIRR) / Financial Management Rate of Return (FMRR) – Pages 42 and 43

\[
\text{Initial Value} = \sum \left[ \frac{C_m}{(1 + \text{Finance Rate})^m} \right]
\]

\[
\text{Terminal Value} = \sum \left[ \frac{C_m}{(1 + \text{Re-investment Rate})^{n-m}} \right]
\]

\[
\text{MIR or FMRR} = \left[ \frac{(\text{Terminal Value} / \text{Initial Value})^{1/n} - 1}{1/(1+d)} \right]
\]

Present Value Formula – Page 45

\[
PV = \frac{CF_1}{(1+d)} + \frac{CF_2}{(1+d)^2} + \ldots + \frac{CF_n}{(1+d)^n}
\]
Net Present Value Formula – Page 47

\[
NPV = CF_0 + CF_1/(1+d) + CF_2/(1+d)^2 + ... + CF_n/(1+d)^n
\]

Profitability Index – Page 48

**Profitability Index = PV of Cash Flows / Investment Cost**

Mortgage Payment for Fixed-Rate Loans – Page 49

**Mortgage Payment = Mortgage Constant \times Loan Amount**

Interest Payment – Page 49

**Interest Payment = Interest Rate \times Beginning Balance**

Principal Payment – Page 49

**Principal Payment = Mortgage Payment - Interest Payment**

Ending Balance – Page 49

**Ending Balance = Beginning Balance - Mortgage Payment + Interest Payment**

Beginning Balance – Page 49
II. RATIO MEASURES OF REAL ESTATE INVESTMENT PERFORMANCE

1. GROSS INCOME MULTIPLIER OR GROSS RENT MULTIPLIER

The Gross Income Multiplier indicates how many times the price/value of the property is greater than the gross income it delivers to its owner. However, because there are two concepts of gross income in the real estate industry, the Potential Gross Income (PGI) and the Effective Gross Income (EGI), there are also two respective multipliers that involve a property’s gross income, the Potential Gross Income Multiplier (PGIM) and the Effective Gross Income Multiplier (EGIM):

The formula for the Potential Gross Income Multiplier (PGIM) is:

\[
PGIM = \frac{\text{Market Price}}{\text{Potential Gross Income}}
\]

This is more meaningful to be calculated on an annual basis and thus the annual Potential Gross Income (PGI) is typically used in this formula. The Potential Gross Income Multiplier indicates how many times the price/value of the property is greater than the potential gross income it can deliver to its owner.

Potential Gross Income (PGI) formula:

\[
PGI = \text{Potential Gross Rental Income (PGRI)} + \text{Other Income}
\]

\[
PGRI = \text{Net Leasable Area} \times \text{Market Rent (per sq. ft.)}
\]

or

\[
PGRI = \text{No. of Units} \times \text{Market Rent (per unit)}
\]

Notice that the Potential Gross Income includes primarily rental income, but it accounts also for any other income that may be produced by the property, such as income from vending machines, laundry room, parking, etc.

**Example:**

Number of Units = 10
Market Rent Estimate (Annual) = $12,500
Other Income (Annual) = $5,000
Market Price = $1,000,000

Therefore,
Potential Gross Income = 10 * 12,500 + 5,000 = 130,000

and
Potential Gross Income Multiplier = 1,000,000/130,000 = 7.69

Thus, in this example, the asking price is 7.69 times greater than the potential gross income produced by the property.

The formula for the Effective Gross Income Multiplier (EGIM) is:

\[
EGIM = \frac{\text{Market Price}}{\text{Effective Gross Income}}
\]

The formula for the Effective Gross Income (EGI):

\[
EGI = \text{Potential Gross Rental Income} + \text{Other Income} - \text{Vacancy & Bad Debt Allowance}
\]

Notice that the Potential Gross Income includes primarily rental income, but it accounts also for any other income that may be produced by the property, such as income from vending machines, laundry room, parking, etc. The vacancy and bad debt allowance accounts for space/units that remain vacant during the year and, as such, do not actually provide any rental income to the landlord, while bad debt allowances cover any rent that is owed during the year but is not paid by the tenants.

**Example:**

Potential Gross Rental Income = $125,000
Vacancy and Bad Debt Allowance (8%) = $10,000
Other Income = $5,000
Market Price = $1,000,000
Therefore,

\[
\text{Effective Gross Income} = 125,000 + 5,000 - 10,000 = 120,000
\]

and

\[
\text{Gross Income Multiplier} = \frac{1,000,000}{120,000} = 8.33
\]

Thus, in this example, the asking price is 8.33 times greater than the effective gross income produced by the property.

2. \textbf{Net Income Multiplier (NIM)}

\[
\text{NIM} = \frac{\text{Market Price}}{\text{Net Operating Income (NOI)}}
\]

As we will see below, the Net Income Multiplier is the reverse of the capitalization rate and denotes how many times the price/value of a property is greater than its Net Operating Income.

Net Operating Income formula:

\[
\text{NOI} = \text{Effective Gross Income} - \text{Operating Expenses} + \text{Recoveries}
\]

Notice that typically \textit{commercial} leases include clauses through which a significant portion of operating expenses is recovered from the tenants. Thus, the third term in the formula above represents the amount of operating expenses that is recovered from the tenants.

Operating expenses are the \textit{sum} of the following expenses:

\begin{align*}
\text{Plus} & \quad \text{Management Fee} \\
& \quad \text{Utilities}
\end{align*}
Example (continuing from previous):

Market Price = $1,000,000  
Effective Gross Income = $120,000  
Operating Expenses = $35,000  
Recoveries = $15,000

Therefore,  
NOI = 120,000 - 35,000 + 15,000 = 100,000

and  
Net Income Multiplier = 1,000,000 / 100,000 = 10

Thus, in this example, the market price is 10 times greater than the NOI produced by the property.

3. OVERALL CAPITALIZATION RATE / INCOME RETURN / ALL-RISKS YIELD

Capitalization Rate = Net Operating Income (NOI) / Market Price

The overall capitalization rate, or simply the capitalization rate, or cap rate, is the reverse of the Net Income Multiplier. It is actually an income return measure and denotes what percent of the property’s value/purchase price is represented by the Net Operating Income produced by the property. In Commonwealth usage is also referred to as All-Risks Yield.
Example (continuing from previous):
Market Price $1,000,000
NOI $100,000
Capitalization Rate $100,000/1,000,000 = 0.10 or 10%

In this example, the capitalization rate indicates that the property provides a 10% income return given its purchase price or asking price.

The capitalization rate is widely used in property valuation theory and specifically in the direct capitalization approach in which the value of a property V can be calculated as:

\[ \text{Property Value} = \frac{\text{Net Operating Income}}{\text{Market Capitalization Rate}} \]

In the formula above, the capitalization rate used is what the appraiser considers as appropriate market capitalization rate given the property’s characteristics and risk level. Note that the higher the investment risk of the particular property, the higher the capitalization rate (in relation to the market average) that needs to be used in the above formula. Note also that average market capitalization rates differ by property type.

Another ratio that relates with the capitalization rate and property valuation within the context of the direct capitalization approach is the Capitalization Factor, which is equal to:

\[ \text{Capitalization Factor} = \frac{1}{\text{Capitalization Rate}} \] or
\[ \text{Capitalization Factor} = \frac{\text{Property Value}}{\text{NOI}} \]

As the second formula above indicates, the capitalization factor is equivalent with the Net Income Multiplier discussed earlier in this book.
Given how it is calculated, the capitalization rate, or cap rate, is a factor that translates property income into an estimate of property value, when property value is not given or the investor has a required income return/capitalization rate and wants to estimate what acquisition price will provide such an income return. This value estimation technique is referred to in the appraisal literature as the direct income capitalization approach, which is widely used especially for a quick and rough estimation of the value of an income producing property. However, its application requires the estimation of market capitalization rates. A more sophisticated income capitalization methodology is the Discounted Cash Flow (DCF) model that takes into account the property’s projected income (both from rents and resale of the property at the end of the holding period) and expenses to estimate the value of the property as the present value of the net cash flow discounted by the investor’s required annual return (income return plus appreciation return) over the holding period.

Within this context, the first step in valuing a property using the capitalization rate is to assess its ability to produce income, and particularly its Net Operating Income (NOI) as explained above. The NOI can be estimated by constructing an operating statement for the subject property.

The second step in estimating the market value of an income producing property using the direct capitalization approach is to estimate the appropriate market capitalization rate. Below we discuss the most commonly used techniques for the estimation of market capitalization rates.

3.1 CAPITALIZATION RATE ESTIMATION TECHNIQUES

The next step in applying the direct income capitalization approach is to determine the appropriate market capitalization rate to be used for the estimation of the value of the subject property. This is necessary because market cap rates vary both across property types and through time. Furthermore, since no single property is exactly the same with another property cap rates vary even across individual properties in a way
that reflects their differences in terms of the factors that affect value.

**Estimating Capitalization Rates Using Comparables**

Within this framework the appropriate capitalization rate is typically determined by analyzing cap rates for sales transactions involving *comparable* properties in the area of the subject property. Subsequently the appraiser needs to “reconcile” these rates by weighting them on the basis of the differences of the comparables from the subject property in terms of location, condition, amenities, income-earning capacity, creditworthiness of tenants, occupancy rate, etc. Obviously, cap rates involving sales of properties that are more similar to the property under consideration will be given greater weighting. Based on the value formula given above capitalization rates for comparable sales can be estimated as:

\[
\text{Capitalization Rate} = \frac{\text{Net Operating Income}}{\text{Value}}
\]

Thus, the capitalization rate for an apartment building that produces an NOI of $16,000 and sold for $200,000 is $16,000/200,000 = 0.08. When estimating market capitalization rates from comparables, extra caution is needed to ensure that the NOI data used for each property are accurate and that they have been estimated in a consistent way across transactions. Once the appropriate market cap rate is determined through the analysis of adequate comparable sales, the property can be valued using that capitalization rate. So, if for example the analyst determines that the appropriate market cap rate is 0.08 then the value of the apartment building in our example that produces an NOI of $21,000 will be:

\[
V = \frac{21,000}{0.08} = 262,500
\]

**The Band-Of-Investment Technique to Estimate Capitalization Rates**
If enough good sales comparables can not be found to come up with a reliable estimate of the appropriate market capitalization approach then a theoretical/ mathematical approach can be followed to come up with an appropriate capitalization rate. The band-of-investment technique is such an approach but it requires the use of a market equity rate for the specific type of property valuated. This rate refers to the equity return that would be required by an investor for investing at the specific property at the given location. The estimation of an appropriate equity rate for a given property is not easy and it has to take into account market required rate return for the risk level that characterizes the property under consideration, given its type, location and idiosyncratic characteristics.

The formula for estimating the appropriate capitalization rate with the band-of-investment technique is the following:

\[ C = (\text{Equity Percentage} \times \text{Equity Rate}) + (\text{LTV Ratio} \times \text{Debt Rate}) \]

Where

- \( \text{LTV} = \frac{\text{Loan Amount}}{\text{Property Value}} \)
- \( \text{Equity Percentage} = 1 - \text{LTV} \)
- \( \text{Equity Rate} = \text{Equity return required by investor} \)
- \( \text{Debt Rate} = \text{Total Return required by lender (see below)} \)

Notice that, in the formula above, LTV stands for loan to value ratio and is calculated as the ratio of the amount of the loan over the value of the property. So for example if the loan amount is 75% of the value of the property then the LTV is 0.75. The equity rate is the percentage of the value of the property that is not financed by loan but by the investor's own money. So if the LTV is 0.75 then the equity percentage is 0.25.

We have already explained what the equity rate is and for the sake of this example we will assume that it is 12%. The debt rate is the total return required by the lender and is NOT the interest rate of the loan, because it also includes and the return of the lenders capital. The debt rate entering the band-of-investment approach is actually
the mortgage constant. The formula for calculating the mortgage constant (MC) is:

\[
MC = \frac{i}{(1 - \left[\frac{1}{(1+i)^n}\right])}
\]

where \(i\) is the loan rate and \(n\) the term of the loan. So for a 20-year loan at an 8% interest rate the mortgage constant would be:

\[
MC = \frac{0.08}{(1 - \left[\frac{1}{(1.08)^{20}}\right])} = 0.101852
\]

Now, we have in our example all the numbers needed to apply the band-of-investment technique to estimate the cap rate:

Capitalization rate = (0.25 x 0.12) + (0.75 x 101852) = 0.03 + 0.076 = 0.106

**Theoretical Approach for Estimating Capitalization Rates**

In theory the capitalization rate is the sum of four components:

1) Expected Inflation
2) Real Return
3) Risk Premium
4) Recapture Premium

By definition, an investment is the commitment of capital in exchange of a monetary benefit, or a return. Investors require a return on the capital invested as a prerequisite for committing capital to a given venture or property. This required return should first provide for the preservation of the purchasing power of invested capital through time. Hence, the first component of required return is expected inflation, so that the purchasing power of invested capital will not decline through time. Ideally, this component is estimated based on inflation rate forecasts, however, many analysts use an average inflation rate over the past five or ten years.
The second component of required return is the real return, which is the true monetary benefit that the investor will gain from committing his/her capital. This is typically estimated as the difference between the rate on government securities and the inflation rate. According to estimates over the past decades this component ranges between 2 and 3 percent.

A property investment is actually an investment in the property’s future income earning capacity. However, there is a lot of uncertainty with this future income earning capacity. For example, tenants may stop paying rent for financial or other reasons; or market rents may decline and new tenants will sign at lower rent; or, even worse, the landlord may not be able to replace tenants vacating the building. In this respect property investments have risk. This risk is the uncertainty associated with the future income stream and the value of the property. Within this context, real estate investors require a risk premium on top of inflation and real return.

The risk premium for a given property depends on the quality of the tenants occupying the property, the length of existing contracts, the property’s occupancy rate, the strength of the property’s location and expectations regarding the prospects of the economy and the local real estate market. For example, the future income stream of an office building with poor-credit tenants is more uncertain and, therefore, more risky than the future income stream of an office building with strong-credit tenants. In addition, the future income stream of an office building in a metropolitan area whose economy is expected to be growing rapidly has less uncertainty and, therefore, less risk than the income stream of an office building in a metropolitan area whose economy is expected to be declining. Likewise, the income stream of a property in a market expected to be facing increasing vacancy rates is more uncertain and more risky than the income stream of a property in a market expected to be facing declining vacancy rates.

Another component of the risk premium attached to real estate investments is liquidity risk, that is, the risk of liquidation or converting the property to cash. Because selling a
piece of real estate at its fair market value is a lengthy and, in many respects, uncertain process, unless a significant price discount is provided, the liquidity risk for property investments is considered high.

There has been a lot of discussion in the professional and academic real estate community about the appropriate risk premium for different property types. Based on historical data it appears that the lower risk premium is associated with apartment investments and the highest with office investments. For the sake of our example we will use a 3% as the risk premium.

Finally, investors require and a recapture premium in the case of property investments, since properties depreciate or lose value through time. Since the value of the property represents the owner’s invested capital, it follows that by the end of the physical life of a building, when its value becomes theoretically zero, the investor loses its capital. The purpose of the recapture premium is to replace this capital loss through time. Thus, if the physical life of a property is 50 years the recapture premium should be 2% on an annual basis. If we assume though that the capital that is recaptured every year is reinvested (sinking fund approach) then a less than 2% recapture rate will be required.

Based on this discussion, let us estimate the capitalization rate as the sum of these four components:

<table>
<thead>
<tr>
<th>Component</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>3.0%</td>
</tr>
<tr>
<td>Real return</td>
<td>2.0%</td>
</tr>
<tr>
<td>Risk Premium</td>
<td>3.0%</td>
</tr>
<tr>
<td>Recapture premium</td>
<td>1.8%</td>
</tr>
<tr>
<td><strong>Capitalization Rate</strong></td>
<td><strong>9.8%</strong></td>
</tr>
</tbody>
</table>

Although this theoretical approach for calculating capitalization rates can provide a solution when sales data for comparable properties are not available, the analyst should contrast the results of such estimates against available market capitalization rate data, even if they refer to the broader market area in which the property operates. Furthermore, the analyst should consult with local real estate investment
professionals and get a range of capitalization rates in the local market within the property is located.

4. **Initial Yield or Going-in Capitalization Rate**

The going-in cap rate or capitalization rate, or initial yield, as is often referred to in Commonwealth usage, is calculated as the ratio of the estimated **annual net operating** income at the date of the transaction (NOI) over the acquisition price of the property. This measure also represents the investor’s income return in the first year of the holding period, but also in subsequent years if NOI remains stable.

**Initial Yield or Going-in Cap Rate = NOI / Acquisition Price**

**Example:**

For the purpose of this example, consider that the property is acquired at $300,000 and that at the time of the transaction it produces a Net Operating Income of $30,000. Therefore we have:

\[
\text{Acquisition Price} = 300,000 \\
\text{NOI}_{TR} = 30,000
\]

Therefore,

\[
\text{Initial Yield or Going-in Cap Rate} = \frac{30,000}{300,000} = 10\%
\]

5. **Terminal Capitalization Rate / Reversion Yield / Exit Cap Rate**

The term Terminal Capitalization Rate referred or Exit Cap Rate, which is referred as Reversion Yield in Commonwealth usage, refers to the rate used to calculate the
resale value of a property by capitalizing the expected net operating income of the property at the end of the planned holding period.

In this sense, and strictly speaking, the analyst needs to forecast what the prevailing local market capitalization rate will be for the property type represented by the property under consideration at the expected time of resale and, then, adjust it accordingly depending on how the property examined deviates from the market average. With an exit cap rate or reversion yield forecast at hand, then the property resale price in the last period \( n \) of the investment horizon can be calculated as:

\[
\text{Resale Price}(n) = \frac{\text{NOI}(n)}{\text{Terminal Capitalization Rate}}
\]

Based on the above formula, if we are given the expected resale price and the expected NOI during the year of resale we can calculate the assumed reversion yield or terminal capitalization rate as:

\[
\text{Terminal Capitalization Rate} = \frac{\text{NOI}(n)}{\text{Expected Resale Price}(n)}
\]

**Example:**

For this example consider that based on the analysts projection's or rental income, other property income and operating expenses it is expected that the NOI in the last year of the holding period will be $100,000. Consider also that based on the anticipated risk profile of the subject property and the anticipated market cap rates in the last year of the holding period it is decided that an 8% terminal capitalization rate or reversion yield can be used to estimate the resale price. Thus:

Expected NOI in the last year of holding period = $100,000

Expected Terminal Capitalization Rate = 8%

Therefore,

Expected Resale Value = \( \frac{100,000}{0.08} = 1,250,000 \)
5. APPRECIATION OR CAPITAL RETURN

The appreciation return measures the percent change in property value. However, in order to estimate the actual return for the investor we need to deduct any expected sales costs. The one-period appreciation return is given by the following formula:

\[
\text{Appreciation Return} = \frac{(V_t - \text{Sales Cost} - V_{t-1})}{V_t}
\]

or

\[
\text{Appreciation Return} = \left[ \frac{(V_t - \text{Sales Cost})}{V_{t-1}} \right] - 1
\]

Where:

\begin{align*}
V_t & = \text{Property market value at time } t \\
V_{t-1} & = \text{Property market value in the previous period } t-1
\end{align*}

You can use either of the two formulas above, as they will produce the same result. The appreciation return is one of the two components of a property’s total return, in the case of income producing real estate. The other component for such properties is the income return or overall capitalization rate. The property’s total expected return is the sum of the appreciation return and the income return.

Example (continuing from previous):

For the purpose of this example, consider that the value of the property that is priced today at $300,000, which represents its fair market value, increases to $330,000 within a year. Thus, for the purpose of applying the appreciation formula, we have:

\begin{align*}
\text{Property market value at time } t \ (V_t) & = \$330,000 \\
\text{Property market value at time } t-1 \ (V_{t-1}) & = \$300,000 \\
\text{Sales cost (5\% of sales price)} & = \$16,500
\end{align*}

Therefore, the one-year appreciation return for this property is:

\[
\text{Appreciation Return} = \frac{(330,000 - 16,500 - 300,000)}{300,000} = 16,500 / 300,000 = 5.5\%
\]
In this example the property would provide an annual appreciation return of 6.7%. If we did not take out sales cost it would indicate a return of 10%, which would be misleading.

6. Total One-Period Return
The total one-period return calculates the total return provided by the property taking into account both income return and appreciation return. It can be calculated as:

\[
\text{Total Return} = \text{Income Return} + \text{Appreciation Return}
\]

or

\[
\text{Total Return} = \frac{(\text{NOI} + V_t - \text{Sales Cost} - V_{t-1})}{V_{t-1}}
\]

Example (continuing from previous):

\[
\begin{align*}
\text{NOI} &= $100,000 \\
V_{t-1} &= $1,000,000 \\
V_t &= $1,100,000 \\
\text{Sales Cost (3%)} &= $33,000
\end{align*}
\]

\[
\text{Total Return} = \frac{(100,000 + 1,100,000 - 33,000 - 1,000,000)}{1,000,000} = 0.167 \text{ or } 16.7\%
\]

This is consistent also with the earlier calculations of capitalization rate or income return of 10% and appreciation return of 10%, which sum up to 20%, thus conforming to the total return formula.

7. Return on Total Capital (ROR)
The Return on Total Capital (ROR) measures the income return of an income producing property with the implicit assumption that there is no debt. It is equivalent actually to the overall capitalization rate or “free and clear” rate of return, as it is calculated as the ratio of NOI over purchase price or total capital invested. If total capital invested is used though, it could be slightly different than the overall
capitalization rate or income return, where the denominator is the purchase price. Total capital invested can be higher than the purchase price, as it may include other acquisition and pre-acquisition costs, such as notary and other governmental fees associated with transfer of ownership, legal due diligence, market studies, feasibility studies, engineering and environment studies, etc.

\[
\text{ROR} = \frac{\text{NOI}}{\text{Total Capital Invested}}
\]

Example (continuing from previous):

\[
\begin{align*}
\text{NOI} &= \$100,000 \\
\text{Total Capital Invested} &= \$1,050,000
\end{align*}
\]

\[
\text{ROR} = \frac{100,000}{1,050,000} = 0.0952 \text{ or } 9.52\%
\]

We used here the total invested capital, which is on purpose assumed greater than the purchase price of $1,000,000 to demonstrate that ROR can be different than the overall capitalization rate.

8. Return on Equity (ROE) / Cash-on-Cash Return / Cash Flow Rate / Equity Dividend Rate / Equity Capitalization Rate

Return on Equity (ROE), referred to also as Cash-on-Cash Return, Cash Flow Rate, Equity Dividend Rate and Equity Capitalization Rate, takes into account borrowing and for this reason is a more often used real estate investment measure, as most real estate investment transactions involve borrowed funds or mortgage loans in order to finance significant portion of investment cost. The formula for ROE is:

\[
\text{ROE} = \frac{\text{Before-Tax Equity Cash Flow (BTECF)}}{\text{Equity Investment}}
\]

The formula for calculating the Before-Tax Equity Cash Flow (BTECF) is the following:

\[
\text{BTECF} = \text{NOI} - \text{Debt Service}
\]
The formula for calculating Investor’s Equity is the following:

\[
\text{Equity Investment} = \text{Investment Cost} - \text{Loan Amount}
\]

Debt Service includes both interest and principal payments. The formula for calculating the Debt Service, given a loan amount, which is borrowed at the fixed annual interest rate of \( i \), amortized over \( n \) years, is the following:

\[
\text{Debt Service} = \text{Mortgage Constant} \times \text{Loan Amount}
\]

\[
\text{Loan Amount} = \text{Loan-to-Value-Ratio} \times \text{Purchase Price}
\]

\[
\text{Mortgage Constant} = \frac{i}{1 - \left(\frac{1}{1+i}\right)^n}
\]

Notice that if we want to calculate monthly mortgage constant then the monthly interest rate \((i/12)\) needs to be used and \( n \) needs to be expressed in number of months.

**Example (continuing from previous):**

\[
\text{NOI} = 100,000
\]

\[
\text{Loan-to-Value Ratio} = 80\%
\]

\[
\text{Interest Rate} = 6\%
\]

\[
\text{Term of the loan} = 20 \text{ years}
\]

\[
\text{Loan Amount} = 0.8 \times 1,000,000 = 800,000
\]

With this information we can now calculate the mortgage constant, the debt service, the Before-Tax Equity Cash Flow the Equity Investment and finally the Return on Equity measure, using the formulas presented above.

\[
\text{Annual Mortgage Constant} = \frac{0.06}{1 - \left(\frac{1}{1.06}\right)^{20}} = 0.087185
\]

\[
\text{Annual Debt Service} = 0.087185 \times 800,000 = 69,748
\]

\[
\text{BTECF} = 100,000 - 69,748 = 30,252
\]

\[
\text{Equity Investment} = 1,050,000 - 800,000 = 250,000
\]
ROE = \frac{30,252}{250,000} = 0.121 \text{ or } 12.1\% 

Thus, in this example, the Return on Equity is equal to 12.1\%.

**9. Payback Period**

Payback period provides the number of years required to recoup the initial cash investment in a project or property. The simplistic formula below can provide an estimate of the payback period, if the anticipated cash flow from the property is expected to be constant through time.

\[
\text{Payback Period} = \frac{\text{Equity Investment}}{\text{Annual After-Tax Cash Flow}}
\]

This measure has several limitations. First, annual-cash flows of a property are seldom constant, especially if it is a large one with multiple tenants. Second, this measure ignores cash flows to the investor after the payback period. Third, it ignores any potential appreciation gains.

In order to apply (15) one needs to calculate the After Tax Cash Flow (ATCF). The formula for calculating ATCF is the following, assuming that borrowing is used:

\[
\text{ATCF} = \text{NOI} - (\text{Taxable Income} \times \text{Tax Rate}) - \text{Debt Service}
\]

Taxable Income in the formula above is calculated as:

\[
\text{Taxable Income} = \text{NOI} - \text{Depreciation} - \text{Interest Payment}
\]

Depreciation methods and rates differ by country and are applied to the purchase price in order to calculate the depreciation amount that will be deducted from NOI in order to calculate taxable income.

**Example (continuing from previous):**
Equity Investment Cost = $250,000
NOI = $100,000
Depreciation Rate = 2%
Interest Payment = Loan Amount * Interest Rate = 0.06 * 800,000 = 48,000
Debt Service = 69,748
Tax Rate = 25%

With this information we can now calculate the taxable income, the ATCF and payback period:

Taxable Income = 100,000 – 0.02*1,000,000 – 48,000
=100,000 – 20,000 – 48,000 = 32,000

Tax payment = 32,000 * 0.25 = 8,000

ATCF = 100,000 – 8,000 - 69,748 = 22,252

Payback Period = 250,000/22,252 = 11.23

Thus in this example, the payback period is 11.23 years.

10. BREAKEVEN OCCUPANCY

The Breakeven Occupancy calculates the occupancy percentage that is needed to pay for operating expenses and debt service. Thus, the formula for calculating this measure is the following:

Breakeven Occupancy = (Operating Expenses + Debt Service) / PGI

where

Debt Service = Mortgage Constant * Loan Amount (see page 22 for formulas)
EGI = Potential Gross Income (see page 7 for formulas)

Example
Annual Operating Expenses = 30,000
Annual Debt Service = 45,000
Annual PGI = 100,000
Therefore,

Breakeven Occupancy = (30,000 + 45,000) / 100,000 = 75,000/100,000 = 75%

It should be noted that when determining a specific buildings breakeven occupancy one needs to take into account the total number of rentable units. For example if the above estimate was involving a multi-housing structure with 5 units, the 75% occupancy rate does not make sense because that would refer to 3 units fully leased and one partially leased. This estimate would suggest that with 3 units leased of the total 5 (occupancy rate 60%) property income would be below breakeven while with 4 units leased (occupancy rate 80%) property income would be above breakeven.

It should be noted that in some references breakeven occupancy is defined only in terms of covering debt service, that is, as the ratio of debt service or mortgage payment over the property’s potential gross income.
III. HOLDING-PERIOD PERFORMANCE MEASURES

1. INTERNAL RATE OF RETURN (IRR)

The Internal Rate of Return, or IRR, is the most commonly used measure for estimating return on real estate investments, because it takes into account the timing of cash flows.

In mathematical terms, the IRR is the rate that sets the Net Present Value (NPV) of the investment's cash flow stream equal to zero. This is represented by the mathematical formula below:

\[
NPV = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_n}{(1+IRR)^n} = 0
\]

Where

- \(CF_0\): Investment cost at time 0, which is the time of purchase
- \(CF_1\): Net cash flow in first period of analysis
- \(CF_2\): Net cash flow in second period of analysis
- \(CF_n\): Net cash flow in LAST period of analysis, which includes and the property’s expected resale price in that period

Solving the above formula with respect to the IRR will provide an estimate of this return measure. However, because the solution of this equation is complex, especially when more than two periods are involved, the IRR calculation is carried out through computer programs. For example, Microsoft Excel includes an IRR formula, which can be used to calculate it once the user supplies the cash-flow stream period-by-period. The IRR is typically calculated using after-tax cash flow of each period, which takes into account all revenues and expenses associated with property ownership.

In analyzing any real estate investment the analyst can calculate either an unleveraged IRR, which assumes that the property is acquired using only the
investor’s money, and a **leveraged IRR**, which takes into account the effect of borrowing on cash flows.

The first cash flow in the NPV formula, which is considered to take place in time 0, CF$_0$, represents the investor’s original investment cost for acquiring ownership of the property. This cash flow may include *not only the property purchase price*, but also other acquisition costs, such as notary fees, other government fees associated with transfer of ownership, potential agency fees, legal fees, etc., and pre-acquisition costs associated with necessary due diligence work, such as engineering studies, market studies, feasibility studies, environmental studies, legal due diligence, etc.

Also notice that the last cash flow CF$_n$ takes into account, and the property’s **anticipated Resale Price**, RP$_n$ in addition to any other income received from the property during that period. This resale price is typically calculated using an exit capitalization rate, which is applied to the property’s Net Operating Income (NOI) at the time of anticipated resale. We will examine in more detail in a following section how this resale price is calculated.

**One -Period IRR**

Solving the IRR equation is easy for just one period, which usually in real estate analysis represents. In this case, we have only two cash flows and so Equation (1) can be written as follows:

$$CF_0 + \left[\frac{CF_1}{(1+IRR)}\right] = 0$$

By transforming this equation we can solve for the one-period IRR, as follows:

$$\frac{CF_1}{(1+IRR)} = -CF_0$$

$$CF_1 / -CF_0 = 1+IRR$$

$$IRR = \left[ \frac{CF_1}{-CF_0} \right] - 1$$

Notice that in the formulas above $-CF_0$ represents a positive number since $CF_0$ represents acquisition/investment cost and is a negative cash flow to begin with. To
demonstrate with numbers how the one period IRR can be calculated let’s consider a rental property that is bought for $100,000, including all acquisition and pre-acquisition costs. The property is sold exactly after one year netting to the investor $120,000. Consider also that during the year the owner receives a rental income of $12,000 and has operating expenses of $4,000. Let’s summarize the figures below:

Acquisition Cost CF$_0$ = $100,000  
Rental Income = $12,000  
Expenses = $4,000  
Net Sales Proceeds = $120,000

Based on the numbers below we can calculate, CF$_1$, the cash flow at the end of the first year when the property is sold:

CF$_1$ = $12,000 - $4,000 + $120,000 = $128,000

Notice that CF$_0$ is negative (-$100,000) and CF$_1$ is positive (+$128,000). By substituting these values in (22) we have:

\[
\text{IRR} = \left[ \frac{$128,000}{-$100,000} \right] - 1
\]
\[
\text{IRR} = 1.28 - 1
\]
\[
\text{IRR} = 0.28 \text{ or } 28\%
\]

**Multiple-Period IRR**

As indicated earlier, solving for the IRR for multiple periods is not that simple and the mathematics become more complex the greater the number of periods over which the investment analysis is performed.

For this reason commonly used spreadsheet programs like Microsoft Excel have automated the calculation of an IRR over several periods. The analyst though still needs to accurately calculate the final cash flow for each period of analysis that is being used by the program in order to calculate the IRR.
At this point we have to emphasize the fact that the cash flows used to calculate the IRR of a real estate investment by definition refer to the future. This means that all revenues and costs that enter the IRR analysis need to be projected to the future.

**Calculating Cash Flows for Each Period**

As indicated earlier the estimation of an IRR involves typically the calculation of the after-tax cash flows of each period. Although in presenting the performance measures in the previous section we have presented how the after-tax cash flow is calculated we are presenting the whole process below:

\[
\begin{align*}
\text{Gross Potential Income} & - \text{Vacancy and Bad Debt Allowance} \\
\text{Plus} & \quad \text{Other Income} \\
\text{Equals} & \quad \text{Gross Effective Income} \\
\end{align*}
\]

\[
\begin{align*}
\text{Gross Effective Income} & - \text{Operating Expenses} \\
\text{Plus} & \quad \text{Recoveries} \\
\text{Equals} & \quad \text{Net Operating Income} \\
\end{align*}
\]

\[
\begin{align*}
\text{Net Operating Income} & - \text{Debt Service} \\
\text{Equals} & \quad \text{Before Tax Cash Flow} \\
\end{align*}
\]

\[
\begin{align*}
\text{Before Tax Cash Flow} & - \text{Tax Payment} \\
\text{Equals} & \quad \text{After Tax Cash Flow} \\
\end{align*}
\]

We have discussed so far how Gross Effective Income, NOI, Tax Payment and Debt Service, are calculated but here are again the formulas for the last two:

\[
\begin{align*}
\text{Tax payment} & = (\text{NOI} - \text{Depreciation- Interest Payment}) \times \text{Tax Rate} \\
\text{Debt Service} & = \text{Mortgage Constant} \times \text{Loan Amount} \\
\end{align*}
\]

It is important here to present the formula for calculating the cash flow for the last period, which needs to take into account not only the cash flow from the income produced by the property as described above, but also the cash flow from the resale of the property. In addition, it has to take into account the payment of the remaining balance of the loan if any borrowed funds were obtained by using the property as
collateral. Thus the formula for calculating the last cash flow of the holding period, \( CF_n \) is:

\[
CF_n = ATCF + Resale Price_n - Selling Expenses - Capital Gain Tax - Loan Balance
\]

Where

Capital Gains Tax = \([\text{Resale Price}_n - \text{Selling Expenses} - \text{Cost Basis}] \times \text{Capital Gain Tax Rate}\)

Cost Basis = \(\text{Purchase Price} + \text{Purchase Expenses} + \text{Capital Expend.} - \text{Cumul. Depreciation}\)

**Example:**

\[
\begin{align*}
\text{ATCF} &= 22,252 \\
\text{Purchase Price} &= 1,000,000 \\
\text{Purchase Expenses} &= 50,000 \\
\text{Resale Price} &= 1,200,000 \\
\text{Selling expenses (2\%)} &= 24,000 \\
\text{Capital Expenditures} &= 0 \\
\text{Cumulative depreciation for 2 years (2\% annually)} &= 2 \times 20,000 = 40,000 \\
\text{Loan Balance at end of 2^{nd} year} &= 775,215 \\
\text{Marginal Tax Rate} &= 0.25
\end{align*}
\]

Thus

\[
\begin{align*}
\text{Cost Basis} &= 1,000,000 + 50,000 - 40,000 = 1,010,000 \\
\text{Capital Gains Tax} &= (1,200,000 - 24,000 - 1,010,000) \times 0.25 \\
&= 166,000 \times 0.25 = 41,500 \\
\text{CF}_2 &= 22,252 + 1,200,000 - 24,000 - 41,500 - 775,215 = 381,537
\end{align*}
\]

**Calculating the Resale Price**

As indicated earlier, calculation of an IRR requires the estimation of the market value (MV) or resale price (RP) of the property under consideration in the last period \( n \) of the investment analysis.

Typically, RP or MV is calculated using the income capitalization approach. According to this approach, a property’s market value and resale price in the last period of the investment analysis \( n \), \( RP_n \), is equal to the ratio of the NOI\(_n\) expected to be produced
by the property in that period over the market capitalization rate expected to be prevailing in that period:

\[
\text{Resale Price}_n = \frac{\text{NOI}_n}{\text{Exit Capitalization Rate}_n}
\]

Please see earlier discussion for further information on how exactly NOI is calculated. In calculating the NOI, in the case of multi-tenant buildings, the Rental Income, RI, in the first period of analysis is calculated as the sum of the rental income received from each tenant, according to the terms of the signed lease contracts with each of them. Note that the rental rate is typically quoted in $ per square meter per month. Given that the contract rate for each tenant is likely different, the formula for calculating the annual rental income for a property with \( i \) tenants at the time of analysis \( t \) is given by (26):

\[
\text{Rental Income}_t = A_1 \times R_{1t} \times 12 + A_2 \times R_{2t} \times 12 + \ldots + A_i \times R_{it} \times 12
\]

Where

- \( A_1 \) = Area (in square meters or square feet) leased by Tenant 1
- \( R_{1t} \) = Rental rate per sq.m. or sq.ft. per month paid by Tenant 1 in year \( t \) according to the terms of the contract
- \( A_i \) = Area (in square meters or square feet) leased by last Tenant \( i \)
- \( R_{it} \) = Rental rate per sq.m. or sq.ft. per month paid by Tenant \( i \) in year \( t \) according to the terms of the contract

Notice that rental income and operating expenses will be available for the period of analysis, which we define as time period 0, but will need to be projected up to period \( n \) in order to estimate the NOI and cash flows during the period of analysis, as well as the Resale Price in the last period \( n \).

Commercial and residential leases have escalation clauses that usually stipulate the rate by which the rent payable by the tenant increases annually. This rate is usually
set equal to Consumer Price Inflation (CPI) in previous 12 months from the stipulated date of annual rent adjustment, plus an increment D, which ranges usually from one to three percentage points. The analyst needs to project how the rental rates for each tenant will increase based on these clauses, in order to build the cash flow stream of the property and estimate the IRR that is likely to be achieved if the property is purchased at a given price. To this aim, reliable CPI forecasts need to be obtained from competent economic forecasting firms. Once such forecasts are available, the generic growth formula can be used to project the rent/square meter/month expected to be received from each tenant one period ahead:

\[ R_{t+1} = R_t \times [1 + \text{Growth Rate (G)}] \]

As indicated, in the case of lease rates, the growth rate G is equal to the sum of observed CPI in previous 12 months, which we will refer to as CPI$_t$, plus the increment D. Thus the rent for each period can be calculated as:

\[ R_{t+1} = R_t \times [1+CPI_t+ D] \]

where

- \( R_{t+1} \) = Projected rent in period \( t+1 \)
- \( R_t \) = Rent in previous period \( t \)
- \( CPI_t \) = CPI forecast in previous 12 months
- \( D \) = percentage increment applied to CPI, as stipulated in lease contract for the particular tenant

The length of period \( t \) in the rent calculation formula is defined as the period stipulated by the lease contract after which a rental adjustment must take place in accordance to the terms of the escalation clause. This period is typically a year, but it is possible for different adjustment periods to be stipulated. For example, some leases may stipulate no rent increases during the first two years of the lease. Lease terms and rent
escalation clauses may differ from lease to lease because of different market conditions prevailing at the time of the signing of the lease, as well as because of the different strength and size of each tenant with which the landlord negotiates. For this reason, it is important to read carefully each lease contract and understand the exact timing and magnitude of the stipulated rent increases, when forecasting the rental income of a multi-tenant building.

In order to demonstrate how the rent projection formula is applied, let’s assume that the rent for Tenant 1 at the time of analysis $t$, $R_{1t}$, is $15/\text{sq.m.}/\text{month}$, that the CPI over the last 12 months, $\text{CPI}_t$, was 3% and that the annual rental adjustment stipulated by the contract is CPI + 2% (which means that $D=2\%$). Given these numbers then we can forecast $R_{1,t+1}$ by using the aforementioned formula:

$$R_{1,t+1} = 15 \times [1 + 0.03 + 0.02] = 15 \times 1.05 = 15.75$$

In order to forecast the rent in two periods ahead, $R_{t+2}$, we can use the following formula:

$$R_{t+2} = R_t \times [1 + \text{CPI}_t + D] \times [1 + \text{CPI}_{t+1} + D]$$

For example, if the inflation forecast for the first period of the investment analysis is 4%, then the rent for Tenant 1 in the second period of the forecast, $R_{1,t+2}$, will be:

$$R_{1,t+2} = 15 \times (1.05) \times (1 + 0.04 + 0.02) = 16.695$$

By extension, assuming that the same annual rent adjustment rule applies for all years in the lease contract, the formula for forecasting the rent for $ n $ periods ahead will be:

$$R_{t+n} = R_t \times [1 + \text{CPI}_t + D] \times [1 + \text{CPI}_{t+1} + D] \times \ldots \times [1 + \text{CPI}_{t+n-1} + D]$$
It is understandable that if the CPI forecast is the **same for all periods** in the investment analysis period then the formula for calculating the rent in \( n \) periods ahead:

\[
R_{t+n} = R_t \times [1 + CPI_t + D] \times [1 + CPI_{t+1} + D]^{n-1}
\]

The other component needed to be projected ahead in order to be able to estimate NOI and cash flows during the period of analysis is operating expenses. In projecting operating expenses forward usually a CPI forecast is used. The analyst can apply the CPI forecast on total operating expenses, unless there is a separate inflation forecast for a particular subcategory of operating expenses. In projecting operating expenses ahead the generic growth formula mentioned for rent can be used, by substituting rent with operating expenses.

**Problems of the Internal Rate of Return Calculation Technique**

**a. The Re-investment Rate Assumption**

One of the potentially problematic assumptions of the leveraged IRR calculation is the reinvestment assumption. In other words, the formula for the calculation of the IRR incorporates the assumption that all positive cash flows in any period are reinvested at the same rate as the calculated IRR. For example, an estimated IRR of 80% implies, implies the expectation that all positive cash flows received at any period will be re-invested immediately at a rate of return of 80%. However, this assumption maybe very unrealistic, if the investor does not have immediately available investment vehicles that will be providing an 80% return. If the actual re-investment rate is lower than the estimated IRR, then the true IRR of the investment is actually lower, depending on how lower is the real re-investment rate from the estimated IRR. The solution to this problem is to estimate a Modified IRR (MIRR) or Financial Management Rate of Return (FMRR), which we discuss in more detail below.
b. Multiple-Solutions

A second important problem of the IRR formula is that it can give multiple solutions depending on the pattern of cash flows that enter the formula. Greer and Farrell (1992) argue that it is possible to have as many IRR solutions as the number of sign reversals (from negative to positive or vice versa) in the cash-flow series used. A typical cash flow pattern used for the estimation of an IRR, has the first cash flow negative, which is the investment cost for the acquisition, but subsequent cash flows can be either positive or negative, depending on revenues, costs, tax payments, loan payments etc. So sign reversals are not uncommon in cash flow patterns for real estate projects. However, multiple sign reversals do not necessarily imply an unreliable IRR estimate, due to potentially multiple solutions, because for many cases of cash-flow patterns with sign reversals there is still one solution.

2. Modified IRR (MIR) / Financial Management Rate of Return (FMRR)

If the investor feels that he/she cannot reinvest all positive cash flows immediately as they are received at an immediately accruing rate of return equal to the estimated IRR then the measure referred to as «Modified IRR» (MIR) by some and as Financial Management Rate of Return (FMRR) by others needs to be used. The FMRR methodology has been developed by Findley and Messner. This measure addresses both potential problems of the IRR estimation process, that is, the reinvestment rate and the multiple solutions.

This estimation methodology can take into account that all positive cash flows are reinvested at a different rate than the IRR of the investment analyzed. Furthermore it eliminates the problem of potentially unreliable estimates due to sign reversals. The MIRR/FMRR methodology allows the use of a reinvestment rate that is different than the IRR. The rate used is generally a savings rate or a government bond rate. However, in the case of experienced real estate investors and investment companies, their real re-investment rate of cash at hand, as has been achieved in the course of their operations should be used, independently of prevailing savings or government
bond rates. The use of the MIRR/FMR methodology is more consistent with reality, as it is rarely possible to reinvest the cash flow from a particular project at exactly the same rate of return, as determined by the IRR formula.

The MIRR/FMRR calculation involves the following three steps:

1. Calculate the present value of negative cash flows incurred in any year during the course of the investment, discounting them at the annual interest rate that you would pay to cover any negative cash flows incurred during the life of the investment (Finance rate). This must be done by estimating the present value of each negative cash flow, taking into account the number of periods that intervene between time 0 and the time each of these cash flows occur, and then summing them up to estimate the Initial Value which will be negative.

2. Calculate the future value at the end of the holding period of all positive cash flows incurred in any year during the holding period of the investment, by growing them at the rate expected to be earned on cash generated by the investment (Re-investment Rate). This re-investment rate can be the government bond rate, or another rate depending on each investor’s access of immediately available investment choices. This must be done by estimating the future value of each positive cash flow taking into account the number of periods that remain until the end of the holding period, and then summing them up to estimate the Terminal Value.

3. Calculate the discount rate that will make the present value of the Terminal Value equal to the Initial Value (in absolute terms as Initial Value has a negative sign) taking into account the number of periods in the investment analysis.

The formulas involved in the three steps of the MIRR/FMRR calculation are the following:
1. Estimate the present value at time 0 of all negative cash flows and sum them up to estimate the Initial Value:

\[ \text{Initial Value} = \sum \left[ \frac{C_m}{(1+\text{Finance Rate})^m} \right] \]

Where

\( m \) = number of periods from time 0 during which each negative cash flow occurs

Finance Rate = Projected cost of capital over the analysis period

2. Estimate the future value at the end of the holding period of all positive cash flows and sum them up to estimate the Terminal Value:

\[ \text{Terminal Value} = \sum \left[ \frac{C_m}{(1+\text{Re-investment Rate})^{n-m}} \right] \]

Where:

\( m \) = number of periods from time 0 during which each cash flow occurs

\( n \) = total number of periods over which the property is analyzed

Re-investment Rate = Realistic re-investment rate as judged by investor, or alternatively, government bond rate

3. Estimate the discount rate that sets the Terminal Value equal to the Initial Value (ignoring the negative sign of the latter) taking into account the number of periods over which the property is analyzed:

\[ \text{Initial Value} = \frac{\text{Terminal Value}}{(1+\text{IRR})^n} \]

Thus:

\[ (1+\text{IRR})^n = \frac{\text{Terminal Value}}{\text{Initial Value}} \]

\[ 1+\text{IRR} = \left( \frac{\text{Terminal Value}}{\text{Initial Value}} \right)^{1/n} \]
and

$$\text{MIRR/FMRR} = \left[ \frac{\text{Terminal Value}}{\text{Initial Value}} \right]^{1/n} - 1$$

Example:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-200,000</td>
</tr>
<tr>
<td>1</td>
<td>150,000</td>
</tr>
<tr>
<td>2</td>
<td>-120,000</td>
</tr>
<tr>
<td>3</td>
<td>520,000</td>
</tr>
</tbody>
</table>

Cost of Capital = 6%

Re-investment Rate = 10%

1. Estimation of Initial Value as the sum of the present value of all negative cash flows discounted at the cost of capital at time 0

Initial Value = -200,000 − \[\frac{120,000}{(1+0.06)^2}\] = -200,000 − 106,799.57 = -306,799.57

2. Estimate the Terminal Value as the sum of the future values at the end of period 3 of all positive cash flows compounded at the presumed re-investment rate of 10%

Terminal Value = 150,000 \(1+0.10\)^{3-1} + 520,000 \(1+0.10\)^{3-3}

= 150,000 \times 1.21 + 520,000

= 181,500 + 520,000 = 701,500

3. Estimate the discount rate that equalizes the Initial Value (without negative sign) with Terminal Value

$$\text{MIRR/FMRR} = \left[ \frac{701,500}{306,799.57} \right]^{1/3} - 1 = 1.317 - 1 = 0.317 \text{ or } 31.7\%$$

Thus, in our example, the expected modified internal rate of return that takes into account the investor’s particular re-investment rate is 31.7\%. 
3. **Present Value (PV)**

As a measure of investment performance, PV is the present value of all cash flows associated with the investment (excluding investment cost, \( CF_0 \)) over its holding period discounted at the investor’s required rate of return. If the PV is higher than investment cost then it means that the expected investment return exceeds the investor’s required rate of return; if it is lower than investment cost, then it means that the investment will provide a return lower than the required return by the investor. The formula for estimating the PV of a cash flow stream discounted at a discount rate or required rate of return \( d \) is:

\[
PV = \frac{CF_1}{(1+d)} + \frac{CF_2}{(1+d)^2} + \ldots + \frac{CF_n}{(1+d)^n}
\]

Where

- \( CF_0 \): Investment cost at time 0, which is the time of purchase
- \( CF_1 \): Net cash flow in first period of analysis
- \( CF_2 \): Net cash flow in second period of analysis
- \( CF_n \): Net cash flow in LAST period of analysis, which includes and the property’s expected resale price in that period

Thus the formula for calculating the last cash flow of the holding period, \( CF_n \) is:

\[
CF_n = ATCF + \text{Resale Price}_n - \text{Selling Expenses} - \text{Capital Gain Tax} - \text{Loan Balance}
\]

**Balance**

Where:

- Capital Gains Tax = \([\text{Resale Price}_n - \text{Selling Expenses} - \text{Cost Basis}] \times \text{Capital Gain Tax Rate}\)
- Cost Basis = Purchase Price + Purchase Expenses + Capital Expend. – Cumul. Depreciation

**Example:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Cost</td>
<td>200,000</td>
</tr>
<tr>
<td>( CF_1 )</td>
<td>150,000</td>
</tr>
<tr>
<td>( CF_2 )</td>
<td>-120,000</td>
</tr>
<tr>
<td>( CF_3 )</td>
<td>520,000</td>
</tr>
<tr>
<td>Discount Rate (d)</td>
<td>12%</td>
</tr>
</tbody>
</table>
Therefore,

\[
PV = \frac{150,000}{1.12} - \frac{120,000}{(1.12)^2} + \frac{520,000}{(1.12)^3}
\]

\[
= 133,928.57 - 95,663.27 + 370,125.73
\]

\[
= 408,391.03
\]

In our example the Present Value of the cash flow stream over the property’s holding period is 574,355, which is significantly higher than the acquisition/investment cost of 200,000, suggesting that the investment’s holding-period return is higher than the investor’s discount rate/required rate of return of 12%.

4. Net Present Value (NPV)

The Net Present Value concept was mentioned earlier in the discussion of the IRR, which was defined as the discount rate that renders the NPV of all the investment’s cash flow equal to zero.

NPV concept is very similar with the Present Value (PV) concept. As a measure of investment performance, the NPV is the net present value of all cash flows associated with the investment (including investment cost, \(CF_0\)) over its holding period discounted at the investor’s required rate of return. If the NPV is greater than zero then it mean’s that the investment return exceeds the investor’s required rate of return; if it is negative then it mean’s that the investment will provide a return lower than the required return by the investor. The formula for estimating the NPV of a cash flow stream discounted at a discount rate or required rate of return \(d\) is the same as the PV formula with the difference that it includes the investment/acquisition cost \(CF_0\):

\[
NPV = CF_0 + \frac{CF_1}{(1+d)} + \frac{CF_2}{(1+d)^2} + \ldots + \frac{CF_n}{(1+d)^n}
\]

Please see PV and IRR sections for the formulas for the calculation of the last cash flow \(CF_n\).
Example:

\[
\begin{align*}
\text{CF}_0 & = -200,000 \\
\text{CF}_1 & = 150,000 \\
\text{CF}_2 & = -120,000 \\
\text{CF}_3 & = 520,000 \\
\text{Discount Rate (d)} & = 12\%
\end{align*}
\]

\[
\begin{align*}
\text{NPV} & = -200,000 + 150,000/1.12 - 120,000/(1.12)^2 + 520,000/(1.12)^3 \\
& = -200,000 + 133,928.57 - 95,663.27 + 370,125.73 \\
& = $208,391.03
\end{align*}
\]

Notice that \( \text{CF}_0 \) must enter with a negative sign because it represents cost and not revenue.

As indicated above in our example the cash flow stream has a significantly higher than 0 NPV indicating that the investment’s return is considerably higher than the discount rate of 12% used.

5. PROFITABILITY INDEX

The Profitability Index (PI) is a metric that property investors can use to evaluate whether the property under consideration meets their return objectives, thus providing a basis for rejecting or further considering an opportunity. The PI shows how many times the present value (PV) of the property’s cash flow stream over the holding period is higher (or lower) than acquisition or investment cost. A Profitability Index greater than 1 indicates that the expected return over the holding period is higher than the investor’s required rate of return, which is used as the discount rate in estimating the present value of the cash flows expected to be produced by the property over the holding period. A PI smaller than 1 indicates that the property’s expected cash flows will not be high enough to provide the minimum return required by the investor. The formula for calculating the PI is the following:
Profitability Index = PV of Cash Flows / Investment Cost

Or

\[ PI = \frac{PV}{CF_0} \]

**Example:**

Investment Cost = 200,000
PV of Cash Flows = 408,391.03

Therefore,

\[ PI = \frac{408,391.03}{200,000} = 2.04 \]

Thus, the Profitability Index in our example is 2.04, which is considerably greater than 1, suggesting that the return of the particular investment is considerably higher than the investor’s required rate of return.

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**III. BASIC MORTGAGE MATHEMATICS**

**Basic Terms and Formulas**

- **Loan to Value Ratio (LTV)** = Loan Amount / Property Value or Purchase Price
- **Equity** = Property Value or Purchase Price – Loan Amount

The following formulas for mortgage constant and mortgage payment apply only to fixed-rate mortgage loans.

**Mortgage Constant** = Interest Rate / \( (1 - \left[1 / (1+\text{Interest Rate})^n\right]) \)
where \( n \) stands for number of periods corresponding to the term of the loan. Attention: Interest Rate and number of periods must refer to the same time unit (months, quarters, etc.)

**Mortgage Payment** = Mortgage Constant \( \times \) Loan Amount  
**Interest Payment** = Interest Rate \( \times \) Beginning Balance  
**Principal Payment** = Mortgage Payment - Interest Payment  
**Ending Balance** = Beginning Balance - Mortgage Payment + Interest Payment  
**Beginning Balance** = In the first year it is equal to the Loan Amount; Subsequent years it is equal to Ending Balance of previous year

**Example:**

<table>
<thead>
<tr>
<th>Property Value</th>
<th>500,000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan-to-Value (LTV) Ratio</td>
<td>60%</td>
</tr>
<tr>
<td>Loan Amount</td>
<td>300,000</td>
</tr>
<tr>
<td>Loan Term in Years</td>
<td>20</td>
</tr>
<tr>
<td>Annual Interest Rate</td>
<td>6.00%</td>
</tr>
<tr>
<td>Annual Mortgage Constant Formula</td>
<td>0.08718</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Annual Payment</th>
<th>Interest</th>
<th>Principal</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300,000.00</td>
<td>26,155.37</td>
<td>18,000.00</td>
<td>8,155.37</td>
<td>291,844.63</td>
</tr>
</tbody>
</table>

**Fixed Annual Mortgage Payment** = 0.08718 \( \times \) 300,000 = 26,155.37  
**Interest Payment Year 1** = 300,000.00 \( \times \) 0.06 = 18,000.00  
**Principal Payment Year 1** = 26,155.37 - 18,000.00 = 8,155.37  
**Ending Balance Year 1**=300,000.00-26,155.37 + 18,000.00 = 291,844.63  
**Beginning Balance Year 2** = Ending Balance Year 1 = 291,844.63
REFERENCES


